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Three Important Advances in Engineering Strength Theories

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There are there great advances in the research on engineering strength theories in the latter half of the 20th Century. The first advance was the development of strength theory from the single-shear strength theory including the Tresca yield criterion and Mohr-Coulomb failure criterion to theoctahedral-shear strength theory; the second one was that from the octahedral-shear strength theory to the twin-shear strength theory; and the third was the theories from the single criteria to the unified strength theory. These three advances are summarized in this paper. It is interesting and useful for researchers to choose an appropriate failure criterion in studying the strength of materials and structures, for engineers to correctly use it and for students to understand strength theory.

Key words failure criterion, single-shear strength theory, octahedral-shear strength theory, twin-shear strength theory, unified strength theory

Strength theories of material under complex stress states are one of the most important constitutive relations for the study and design of engineering structures[1-6]. Generally, our understanding of strength of material is the uniaxial strength of material under uniaxial tension or uniaxial compression. For example, the uniaxial tension strength for metallic materials and uniaxial compression strength for concrete and rock are used. They are one-dimensional strength of material that can be obtained from experiments. Most of materials in engineering structures, however, are acted under two-dimensional stresses or three-dimensional stresses (these two stresses are called complex stress states). The determination of strength of material under complex stress states is a complex and difficult problem, which is a two-dimensional or three-dimensional stress problem.

The complexity is due to three factors. The first one is its difficulty for producing a two-directional or three-directional load equipment. The second is that we can only get some parts of strength curves or strength surface of material under some combinations of complex stresses. The one-dimensional strength of material is only a point of these curves or surface, while the last one is that we cannot obtain all kinds of strength of materials under various combinations of complex stress experiment even though we have complex stress experiment equipment, because the combinations of two-dimensional or three-dimensional strength are infinite. We need strength theories to predict and analyze the strength of material and structures under different combinations of stress.

Due to these characteristics in the research of strength theory, there is a slow progress in strength theory though its research is so important[1-6].

Theoretical research in strength theory has a connection with experimental research, while the common understanding in the field expects the theoretical research and experimental research in strength theories relatively to keep some independence.

The research of engineering strength theory of materials under complex stress states bears characteristics of diversification. Strength theories were summarized in literatures[1-6].

1 Research of Complex Stress Experimental Facilities

The development of strength theories is closely associated with that of the experimental technology for testing materials under complex stress states. A considerable account of triaxial stress tests was done in the 20th Century. The first researchers include Foppl (1900), von Karman (1911) and Böker (1915); von Karman and Böker are supervised under L, Prandtl.

There are two kinds of triaxial test. In most laboratories, for triaxial tests cylindrical rock and soil specimens are load with an axial stress $\sigma_z = \sigma_1$ (or $\sigma_z = \sigma_3$), and a lateral pressure $\sigma_2 = \sigma_3$ (or $\sigma_2 = \sigma_1$); both can be varied independently, but always $\sigma_2 = \sigma_3$ (or $\sigma_2 = \sigma_1$). Today such tests are done in all rock mechanics and soil mechanics laboratories. This kind of test, unfortunately, is usually called triaxial test, although it involves only very special combinations of triaxial stress, a special plane in stress space. It cannot give any states of complex stress that can be independently controlled in three directions. It is better to refer to this test as the confined compression test (or false triaxial test), since it is a compression test with a confining lateral pressure. But as yet, we have not a machine that can produce any combinations of polyaxial stresses and compose them freely.

In 1914, Böker retested the type of marble used by von Karman in a confined pressure test in which the lateral pressure was the major principal stress. The corresponding Mohr's envelope did not agree with von Karman's (in von Karman's tests, the axial pressure exceeded the lateral pressure). This means that the Mohr-Coulomb criterion could not fit the data adequately in the range of low hydrostatic pressure, although the more general hexagonal pyramid criterion is not ruled out[2]. It is evident that the confined compression test is not capable of proving that the intermediate stress is of no influence on the failure criterion.

Another is seldom true tri-axial test, in which all three principal stresses can be varied independently[2,7].

In the 1960s, a great amount of effort was dedicated to the development of true-triaxial testing facilities, and the facilities were then used to test engineering materials. Some representative efforts were seen on rocks at Tokyo University and others, on soil at Cambridge University, Karlsruhe University and Kyoto University and others, and on concrete in France, Germany, UK and the United States. This situation was mainly due to the researches of the strength of container structure and concrete pressure vessel in nuclear power plant. True triaxial test facility for metallic material has not come into being yet.

Mogi's persistent effort revealed that rock strength varied with the intermediate principal stress σ_2, which was quite different from what had been depicted in the conventional Mohr-Coulomb theory. The study was further extended[8] to a understanding that the σ_2 effect had two zones: the rock strength kept on increase, when σ_2 built up its magnitude from zero, until reaching a maximum value; beyond that, the rock strength decreased with the further increase of σ_2. Xu and Geng also pointed out that varying σ_2 only, while keeping the other principal stresses σ_1 and σ_3 unchanged, could lead to rock failure, and this fact could also be attributable to inducing earthquakes. Michelis indicated that the effect of intermediate principal stress is the essential behavior of materials[7].

The other is tension-compression true triaxial test facility, which is produced by the Third Research Institute of Army of China[9,10]. Tongji University has done some researches in soil test. Dalian Science and Technology University and Tsinghua University have conducted a large number of true triaxial tests for concrete. The true triaxial test results have been summarized in literatures[6,11].

2 From Lower Bound to Upper Bound

A lot of strength theories and criteria were presented after Mohr. The proposed criteria and material models in 20^{th} Century are too much that it is difficult to classify. Fortunately, a fundamental postulate concerning the yield surfaces was introduced by Drucker (1951) and Bishop-Hill (1951) with the convexity of yield surface determined. The convex region and its two bounds are most interesting. One method we used of representing these theories is to use the principal shear stresses τ_{13}, τ_{12}, τ_{23} and the normal stress σ_{13}, σ_{12}, σ_{23}, acted on the same section that the shear stress is acted respectively. Strength theories may be divided into three kinds as follows.

The single-shear strength theory is proposed by Tresca in 1864 and by Mohr in 1900. The former is a special case of the latter. The octahedral-shear strength theory was achieved during the period from 1904 to 1952. The single-shear strength theory developed into the octahedral-shear strength theory is the first significant advance of strength theory of materials under complex stress states in the 20^{th} century. The former only considers the two of three principal stresses, and the latter averages the principal stresses. A lot of octahedral-shear strength theories were proposed during the period from the 1970s to the 1990s. Shen summarized the octahedral-shear strength theories and named them tri-shear strength theories. They are various kinds of curve-type criteria that are situated between the upper bound and the lower bound of convex theory as shown in Fig. 1[11-14].

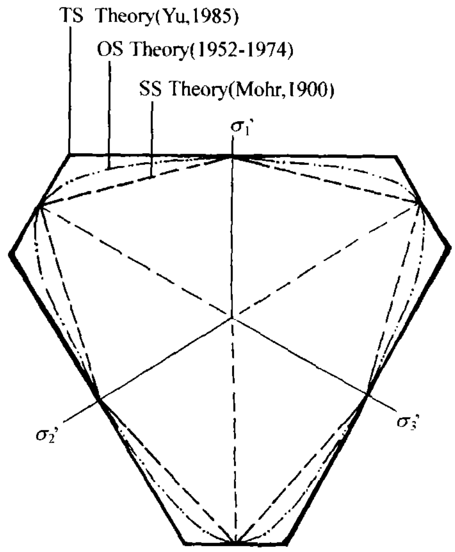

Fig. 1 Limiting loci of SSS, OSS and TSS theories.

Single-shear strength theory is the lower bound of all convex theories. But dose the upper bound exist? If it does, what is it? There was not a definite realization before the 1980s, and these problems have not solved even in theory. All these need proposing new ideas to theoretical conceptions and methods of establishing the mathematical modeling. The second important advance of strength theory is the establishment of the twin-shear strength theory. The twin-shear yield criterion for metallic materials was proposed by Yu in 1961, and the generalized twin-shear strength theory was proposed by Yu, He and Song in 1985[15]. The twin-shear yield criterion is a special case of the generalized twin-shear strength theory, which became the upper bound of convex theory. From the lower bound of 1900 to the upper bound of 1985, it had been 85 years.

The key to solving this problem is that we must break the frame of using a single formula to build a new mechanical model and a new mathematical modeling. Yu first proposed a twin-shear mechanical model and the method that uses two formulas to establish the mathematical modeling in the research of engineering strength theories. The twin-shear strength theory was given in 1985. But there had been 25 years from the twin-shear yield criterion for metallic materials in 1961 to the twin-shear strength theory for rock and soils in 1985. Now, the twin-shear yield criterion and the twin-shear strength theory have been became a paragraph in more than 22 kinds of "Mechanics of Materials" and over 15 kinds of "Engineering Mechanics" and "Plasticity" textbook in China. In reverse, we can feel the advance from the lower bound of the single-shear strength theory to the upper bound of the twin-shear strength theory may be very slow, and the efficiency may be very low. Professors Shen, Jiang, Zhang and Zhao et al. have taken the twin-shear strength theory into the researches of theoretical soil mechanics[16], nonlinear finite-element analysis[12], rock and soil plasticity[17] and metal forming, etc.

Fig.1 shows the limit loci of the single-shear strength theory, the twin-shear strength theory and the octahedral-shear strength theory in devitoric plane. Either the shapes or the sizes of these three limit loci are different from one another. Two advances unceasingly enlarge the region of the limit loci. Their mathematical expressions stepped from linear to non-linear, and then to linear. It was also the processes that the mathematical expressions were developed from simple to complex, and then back to simple.

3 From Single Strength Theory to Unified Strength Theory

The present engineering strength theories can only be used for a certain kind of materials. A united strength theory was proposed in the former Soviet Academy of Science in the 1940s. It was ever regarded as the highest achievement of strength theories by socialism-camp countries before 1970s. It only, however, offered a selection in two strength theories that had been existed. The research of the unified strength theory is another important problem of researches on engineering strength theory.

Under the support of National Natural Science Foundation, a new unified strength theory was proposed in 1991[18,19]. All components of stress are taking into account in the unified strength theory.

Unified strength theory has a unified mechanical model and unified mathematical expressions. It can be adopted for metallic materials, rock, soils, concrete and polymer materials. The single-shear strength theory, the twin-shear strength theory and all criteria between them can be deduced from the unified strength theory. Interesting readers may be referred to a summary article[7] and a new monograph[20].

The limit loci of the unified strength theory on de-

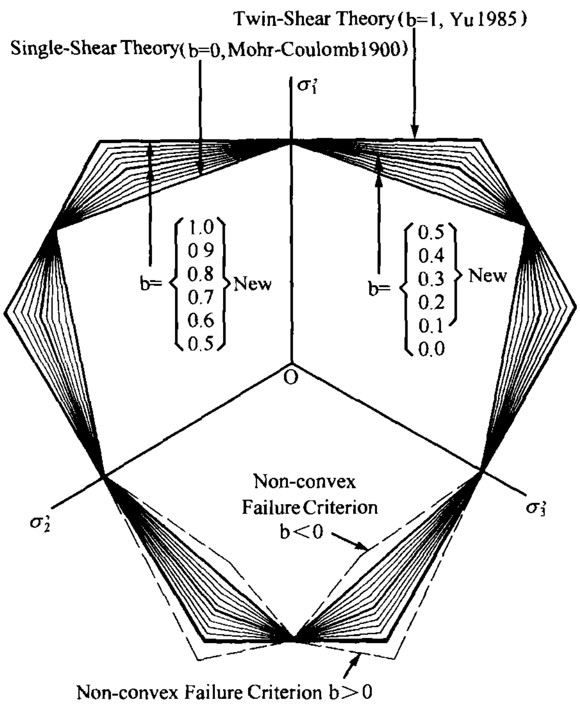

*Fig. 2 **Variation of the unified strength theory on the deviatoric plane** (**Yu 1992, 1994**).*

viatoric plane not only covers the all region of convex theory, but also extends convex theory to non-convex theory. Non-convex theory can been deduced from the unified strength theory, and it has not been studied.

Unified strength theory is the continuous development of the twin-shear yield criteria (1961) and the twin-shear strength theory (1985). The theory encompasses various classical strength theories as special cases and establishes the quantitative relations. It forms a system of strength theories. The limit loci of the unified strength theory covers the all region and can be used for many kinds of materials. It can also be used in the analytic solutions of structural strength problems conveniently because of its linear mathematical expression. The detailed discussions of strength theories can be founded in an article published in Applied Mechanics Reviews [7] and two monographs [6, 20]. The advance of strength theories for concrete can be seen in the monograph[21]. This work was supported by the National Natural Science Foundation of China.

4 Conclusion

Three significant advances of engineering strength theories of materials under complex stress states in the 20th century are summarized in this paper, namely the development from the single-shear strength theory to octahedral-shear strength theory, the octahedral-shear strength theory developed into the twin-shear strength theory, and the development from the single strength theory to the unified strength theory. The latter two were achieved under the support of National Natural Science Foundation of China. They were selected an outstanding achievements by the National Natural Science Foundation of China at the time when it had been founded for 10 years. These advances of strength theory are meaningful and useful for students and graduate students to understanding the strength theory, for engineers to correctly use it and for researchers to choose an appropriate failure criterion in studying the strength of materials and structures.

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dards for toxicopathology evaluation were instituted under the harmonization of multination. Besides holding of anniversary meetings and publishing of periodical journals in Japan, Seminar on experimental pathology histotechnology has been held, which particularly concentrates on resolving technique problems such as sample preparation, section staining and microscopy examination, in order to increase the research level of toxicological pathology[10].

Recently, toxicopathology attracts much attention and is considered as playing a key role either in the traditional and conventional toxicity assessment or in the modern toxicity mechanism studies with advanced techniques[11, 12]. To catch up with the international development as soon as possible, therefore, we toxicopathologists should realize the important task, be farseeing, surefooted and innovative, try to work together to improve our domestic drug safety evaluation to a new height.

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